Partial differencial Equations

* **partial derivatives:** → Let $z = f(x,y)$ be a function of two Variables 'x' and 'y'.

→ if we keep 'y' as Constant and vary 'x' alone, then z is a function of 'x' only. The derivatives of 'z' with respect to 'x', treating 'y' as Constant, called partial derivative of z with respect to 'x' and denoted by one of the Symbols:

→ $\frac{\delta z}{\delta x}, \frac{\delta f}{\delta x}, f_x(x,y), D_x f.$

→ $\frac{\delta z}{\delta y}, \frac{\delta f}{\delta y}, f_y(x,y), D_y f.$ → derivative of z with respect to y, keeping 'x' as Constant.

→ Also, $\frac{\delta}{\delta x}\left(\frac{\delta z}{\delta x}\right) = \frac{\delta^2 z}{\delta x^2}.$

→ $\frac{\delta}{\delta y}\left(\frac{\delta z}{\delta x}\right) = \frac{\delta^2 z}{\delta y \delta x}.$

Q. find the first and second partial derivative of $z = x^3 + y^3 - 3axy$.

Sol^n ⇒ $z = x^3 + y^3 - 3axy$ — we have.

⇒ $\frac{\delta z}{\delta x} = 3x^2 + 0 - 3ay(1) = 3x^2 - 3ay$

⇒ $\frac{\delta z}{\delta y} = 0 + 3y^2 - 3ax(1) = 3y^2 - 3ax.$

Also, $\frac{\delta^2 z}{\delta x^2} = \frac{\delta}{\delta x}(3x^2 - 3ay) = 6x.$

$\frac{\delta^2 z}{\delta y^2} = \frac{\delta}{\delta y}(3y^2 - 3ax) = 6y$